ch 1 : Electric Charges & Fields.

→ Electrostatics :- It is a branch of physics which deals with the study of electric charges at rest.

→ Frictional Electricity :- The electricity acquired by a body due to friction is known as frictional electricity.

→ When a glass rod is rubbed with silk, glass rod gets a +ve charge & silk gets -ve charge due to the transfer of ēs from glass rod to silk.

Egs :-

+vely charged body	-vely charged body
hair	plastic
glass rod	silk
wool	polythene
Nylon	Rubber.

Q1. Electrostatic experiments donot work well in humid days. why?

ans. Water molecules are present in the atmosphere on humid days. Water is a good conductor of electricity. So the charges will be easily discharged through the water molecules to the earth.

Q2 A glass rod can be charged by rubbing it with silk. Can a Cu rod be charged like this? Why?

ans : No.
Cu is a conductor of electricity.

Q3. Vehicles carrying inflammable materials usually have a metallic rope touching the ground. why?

ans (or)

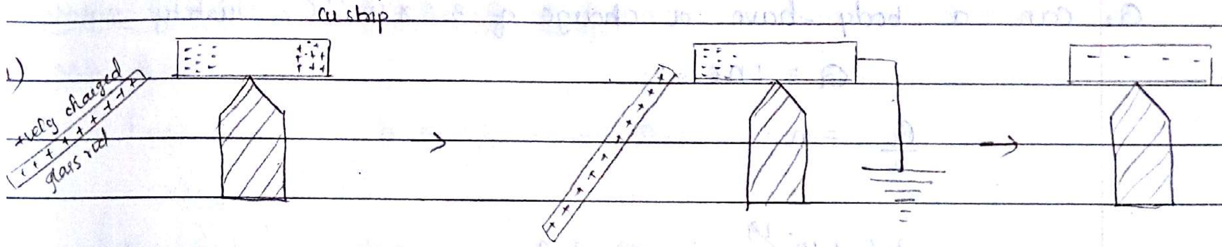
Tyres of aircraft is made slightly conducting. Why?

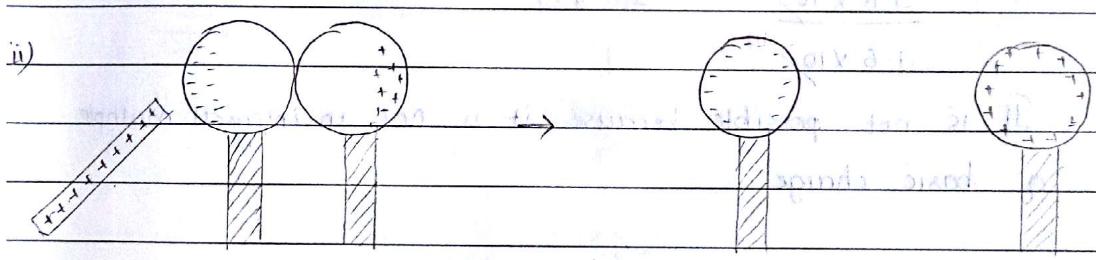
ans. For the easy transfer of accumulated charges to the earth & thus we can avoid an explosion.

Q4. When a glass rod is rubbed with silk, glass rod gets a +ve charge. Is there any transfer of mass from

i) Silk to glass rod
ii) glass rod to silk
i) No
ii) Yes Because of the transfer of $\bar{e}$s.

Electro static Induction :- The electrification (charging) of a body by the presence of another charged body is known as electro static induction





Charge & its properties :- Charge is a characteristic prop. of a body like mass by which it can exert a force on another body having the same prop.

→ There are 2 types of charges, +ve & −ve. The basic charges are $\bar{e}$ & proton

$1\bar{e} = -1.6\times10^{-19}\,C$

$1p = +1.6\times10^{-19}\,C$

→ Like charges repel & unlike charges attract each other

→ Charge is quantised − The tot. charge of a body is an integral multiple of the basic charge. ie

$\boxed{Q = \pm n\bar{e}}$, where $n = 1, 2, 3 \cdots$

& $\bar{e} = 1.6\times10^{-19}\,C$

→ Charge is conserved − Charge can neither be created nor be destroyed but it can be transferred from one body to anoth

→ Charge is additive ie the tot. charge of the body is the sur of the individual charges.

Q1. Calculate the no: of ēs in one Columb of charge.

$$Q = ne$$

$$1.6 \times 10^{-19} = \cancel{n}\,\cancel{\times}\, e$$

$$e = \frac{1}{1.6 \times 10^{-19}}$$

$$= \frac{10^{19}}{1.6} \Rightarrow 6.2 \times 10^{18}$$

Q2. Can a body have a charge of 3.6×10^{-19} C. Justify.

$$Q = \pm ne$$

$$\frac{Q}{e} = n$$

$$\frac{3.6 \times \cancel{10^{-19}}}{1.6 \times \cancel{10^{-19}}} = 2.2$$

It is not possible because it is not an integral multiple of basic charge

Coulombs' Law :- The law states that the electrostatic force of attraction or repulsion b/w 2 charges at rest is directly proportional to product of the charges & inversely proportion-al to the sq. of the distance b/w them.

Consider 2 charges, q_1 & q_2 separated by a dist. r

$$q_1 \longleftarrow r \longrightarrow q_2$$

Then,

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}}$$

where ε_0 is a constant called the permitivity of free space.

$\varepsilon_0 = 8.85 \times 10^{-12} \; C^2/Nm^2$

& $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \; Nm^2/C^2$

If the 2 charges are kept in a medium, then $F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$

where ε is the permitivity of that medium.

Relative Permitivity (Dielectric Constant) :-

We have,

$$F_{vac} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \rightarrow ①$$

$$F_{med} = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \rightarrow ②$$

$$\frac{①}{②} \Rightarrow \frac{F_{vac}}{F_{med}} = \frac{\frac{1}{\cancel{4\pi}\varepsilon_0} \cancel{\frac{q_1 q_2}{r^2}}}{\frac{1}{\cancel{4\pi}\varepsilon} \cancel{\frac{q_1 q_2}{r^2}}}$$

$$\frac{F_{vac}}{F_{med}} = \frac{\varepsilon}{\varepsilon_0}$$

$$\boxed{\frac{F_{vac}}{F_{med}} = \varepsilon_r}$$ where $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ called the relative permitivity of the medium.

The Relative permitive of a medium can be defined as the ratio of permitivity of that medium to the permitivity of free space.

(or)

The relative permitivity of a medium can be defined as the ratio of force b/w 2 charges kept at a distance in vacuum to the force b/w the same charges kept at the same distance in that medium.

(wrt Electrostatic force)

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$② \Rightarrow F_{med} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}$$

Note :- Relative permitivity of air (free space), $\varepsilon_r = 1$

1 Coulomb :-

We have $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

Let $q_1 = 1C$, $q_2 = 1C$ & $r = 1m$

Then $F = 9 \times 10^9$ N

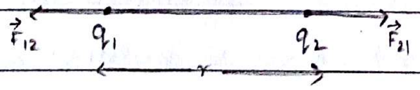
imp ∴ 1 Coulomb is that charge which when placed at a distance of 1m in free space from an equal & similar charge produces a force of 9×10^9 N b/w them.

Coulomb's law in Vector form :-

We have $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

In vector form,

$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$ where $\hat{r}$ is the unit vector in the direction of force.



Force on q_2 by q_1 , $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$ $\hat{r}$ is along the dir... of F. $\vec{F}$ is from 1 to 2

Force on q_1 by q_2 , $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$

The only diff b/w $\hat{r}_{12}$ & $\hat{r}_{21}$ is that they are oppositely directed

ie $\hat{r}_{12} = -\hat{r}_{21}$

$$\therefore \boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

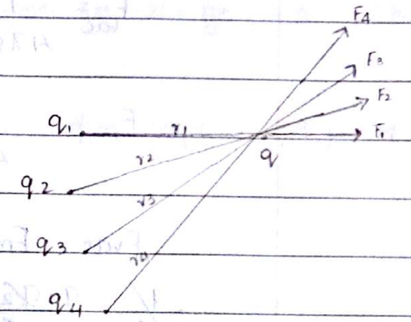
From the above result it is clear that

i) Newton's 3rd law of motion holds good in Coulomb's law

ii) A pair of charges forms an action-reacⁿ pair

Super position principle :- The principle states that when a no. of charges are interacting, the resultant force acting on a charge is equal to the vector sum of forces produced by individual charges.

ie $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} \ldots$



$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q}{r_1^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2 q}{r_2^2}$$

$$F_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_3 q}{r_3^2}$$

$$F_4 = \frac{1}{4\pi\varepsilon_0} \frac{q_4 q}{r_4^2}.$$

ie $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} \ldots$

Numericals

Q. 2 charges -2×10^{-12} C & 3×10^{-12} C are separated by a dist. of 4mm. calculate the Electrostatic force b/w them.

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$F = 9\times10^{9} \times \frac{-2\times10^{-12} \times 3\times10^{-12}}{(4\times10^{-6})^2}$$

$$\frac{9\times10^{9} \times -2\times3\times10^{-24}}{4\times4\times10^{-6}}$$

$$\frac{9\times10^{9} \times -6\times10^{-18}}{4\times4}$$

$$\frac{-54\times10^{-9}}{4\times4\times10^{-3}} = -13.5 \times 10^{-9}\ \text{N}$$

13.
4)54
4
14
12
20

$$\frac{-13.5\times10^{-9}}{4\times10^{-6}} = -3.37\times10^{-9}\ \text{N}$$

Q. 2 charges q_1 & q_2, ~~are~~ separated by a dist. r kept in vacuum. A dielectric medium of dielectric constant, K is inserted b/w the charges & the dist b/w them is doubled. Find the value of K if the force b/w the charges remains the same.

$$F_{vac} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$\varepsilon = \varepsilon_0 K$

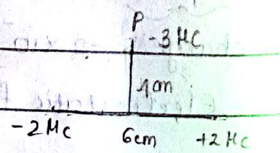
$$F_{med} = \frac{1}{4\pi\varepsilon_0 K} \frac{q_1 q_2}{(2r)^2}$$

$F_{vac} = F_{med}$

$$\frac{1}{\cancel{4\pi\varepsilon_0}} \frac{\cancel{q_1 q_2}}{\cancel{r^2}} = \frac{1}{\cancel{4\pi\varepsilon_0} K} \frac{\cancel{q_1 q_2}}{4\cancel{r^2}} \Rightarrow 1/4K = 1$$

or $K = \frac{1}{4}$

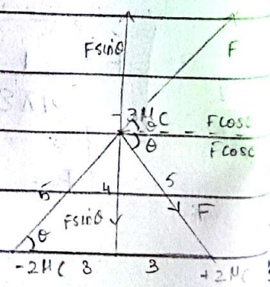
Q3 2 charges $-2\mu C$ & $+2\mu C$ are seperated by a dist. of 6 cm as shown in fig. find the force acting on the charge $-3\mu C$ kept at point P.



$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{25 \times 10^{-4}}$$

$$= \frac{540}{25} \text{ N}$$



$F_{tot} = F\cos\theta + F\cos\theta$

$= 2F\cos\theta$

$\cos\theta = \frac{3}{5}$

$$2 \times \frac{\overset{108}{\cancel{540}}}{25} \times \frac{3}{\cancel{5}}$$

$= 25.9$ N

Electric Field :- The space around a charge where electrostatic force is experienced is called Electric field.

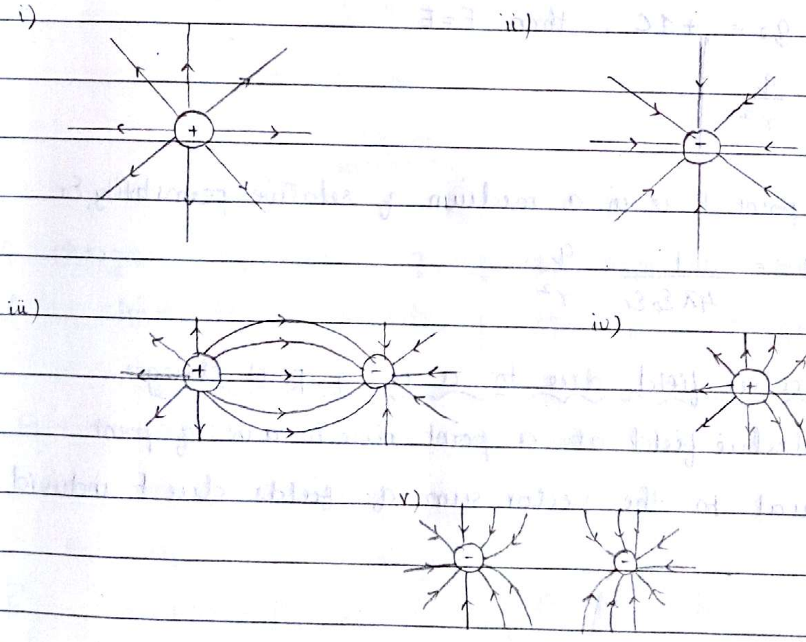
Intensity of Electric field (E) :- Intensity of electric field at a point can be defined as the force experienced by a unit positive charge (+1C) kept at that point.

If a +ve test charge, q, experiences a force, F when kept at a point, then the intensity of electric field at that point,

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

E is a vector quantity & its unit is N/C

Electric field lines :- They are the imaginary lines or curves, tangent to which at any point gives the direction of electric field at that point.



Properties of Electric Field lines :-

→ They start from +ve charge & ends at -ve charge.

→ They donot form closed loops becauses charges have separate existance.

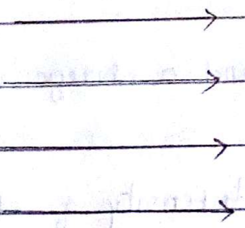
→ 2 field lines will never intersect

. If they intersect at a point, there will be 2 directions for the field which is impossible.

→ closer the field lines, greater the intensity of electric field.

→ They expand sidewise

→ Uniform electric field is represented by equidistant $\parallel^{l}$ lines.



Intensity of electric field due to point charge :-

Consider a point P at a dist. r from a point charge q

q ——— P +1C

⟵ r ⟶

We have, $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

Here $q_1 = q$

Intensity of electric field at P is the force experienced by a unit positive charge kept at P.

ie If $q_2 = +1C$, then $F = E$

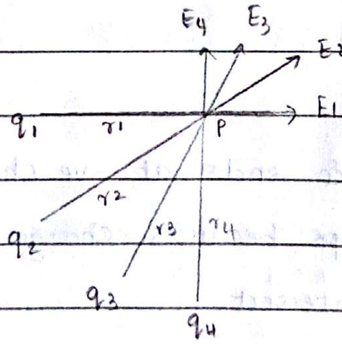
$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}$$

Note :- If the point P is in a medium of relative permittivity, ε_r

Then, $E_{med} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q}{r^2}$

Intensity of electric field due to a no: of point charges

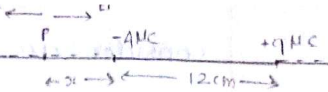
Intensity of electric field at a point due to a no: of point charges is equal to the vector sum of fields due to induvi-ual charges.



$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4} \ldots$

Q. 2 charges $-4\mu C$ & $+9\mu C$ are separated by a dist. of 12 cm. Fi[nd] the point on the line joining these 2 charges where the electric field is 0.

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{4 \times 10^{-6}}{x^2}$$



$$E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{9 \times 10^{-6}}{(12+x)^2}$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}} \times \frac{4 \times \cancel{10^{-6}}}{x^2} = \cancel{\frac{1}{4\pi\varepsilon_0}} \times \frac{9 \times \cancel{10^{-6}}}{(12+x)^2}$$

$$\frac{4}{x^2} = \frac{9}{12^2 + x^2 + 24x}$$

$$4(12^2 + 24x) = 9$$

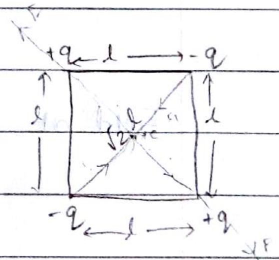
$$\frac{4}{x^2} = \frac{9}{(12+x)^2} \quad \text{(Taking root on both sides)}$$

$$\frac{2}{x} = \frac{3}{12+x}$$

$$24 + 2x = 3x$$

$$x = \underline{\underline{24\,cm}}$$

A charges, +q, +q, -q, -q are kept at the corners of a square of side l. Find the intensity of electric field at the centre of the square.



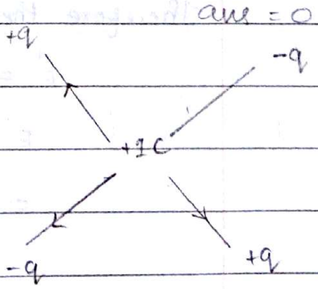
$E_1 = E_2 = E_3 = E_4$ (in magnitude)

$-E_1 = E_3$

$-E_2 = E_4$

$$\vec{E} = E_1 + E_2 + E_3 + E_4$$

$$\cancel{E_1} + \cancel{E_2} - \cancel{E_1} - \cancel{E_2} = \underline{\underline{0}}$$



Electric dipole :- A pair of equal & oppo. charges separated by a small dist is called an electric dipole

Electric dipole moment (P) :- It's a quantity which indicate the ~~sp~~ strength of a dipole. It can be measured as the product of magnitude of the charge & the dist. b/w the 2 charges

Consider an electric dipole consisting of 2 charges, $-q$ & $+q$ separated by a small dist $2l$

$-q$ —— $+q$
$\leftarrow 2l \rightarrow$

Then electric dipole moment, $\boxed{\vec{p} = q \times \vec{2l}}$

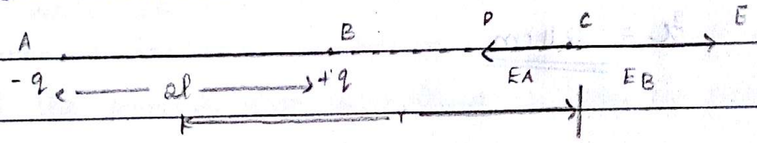
→ Dipole moment is a vector quantity & is directed from -ve charge to +ve charge

→ Its unit is Cm

Intensity of electric field due to a dipole :-

i) At a point on the axial line.

Consider an electric dipole consisting of 2 charges, $-q$ & $+q$, separated by a distance $2l$ & kept at points A & B resp. C is a point at a distance r ~~on~~ from the centre of the dipole on its axial line.



Electric field at C due to the charge $-q$,

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}, \text{ along CD}$$

Field at C due to the charge $+q$,

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2}, \text{ along CE}$$

Therefore the resultant field at C,

$$\vec{E} = \vec{E_A} + \vec{E_B}$$

$$E = E_B - E_A \quad (\because \text{since } E_B > E_A)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2(r+l)^2}\right]$$

$(a+b)(a-b) = a^2 - b^2$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+2rl+l^2-r^2+2rl-l^2}{(r^2-l^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2\times 2rl}{(r^2-l^2)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{2\times 2rl \times q \times r}{(r^2-l^2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{2pr}{(r^2-l^2)^2}$$, where $p = q \times 2l$, the dipole moment.

∵ r is very much greater than l, l^2 can be neglected (charges are separated by very small dist l.)

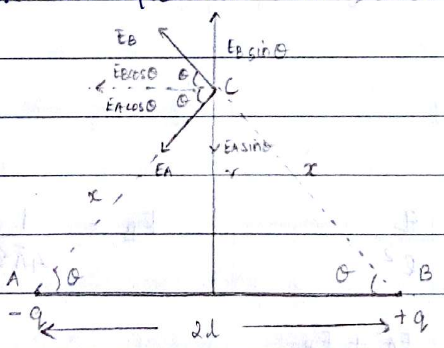
$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{2pr}{r^4}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \times \frac{2p}{r^3}}$$

→ The resultant electric field is directed along the direction of dipole moment.

ii) At a point on the equatorial line :-

C is a point on the equatorial line at a dist. r from the centre of the dipole.



Field at C due to a charge $-q$,

$$E_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$$

Field at C due to a charge, $+q$,

$$E_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$$

$$\therefore \quad E_A = E_B$$

The resultant field at C, $\vec{E} = \vec{E_A} + \vec{E_B}$

The rectangular components of E_A & E_B are as shown in the fig. $E_A \sin\theta$ & $E_B \sin\theta$ are equal & oppo. & cancel each other

Therefore the resultant field at C,

$$\vec{E} = \vec{E}_A \cos\theta + \vec{E}_B \cos\theta$$

$$\vec{E} = 2E_A\cos\theta \qquad (\because E_B = E_A)$$

$$2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} \times \frac{l}{x}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3} \qquad x = (r^2+l^2)^{1/2}$$

$$x^3 = (r^2+l^2)^{3/2}$$

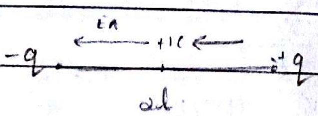
$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+l^2)^{3/2}}$$

Since $r >> l$, l^2 can be neglected.

$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}}$$

→ The direction of electric field is oppo. to the direction of dipole moment.

Q. An electric dipole consists of 2 charges $-q$ & $+q$ separated by a dist $2l$. Calculate the magnitude & direction of electric field at the centre of the dipole.



$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2} \qquad E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$$

$$E = E_A + E_B$$

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{l^2} + \frac{1}{l^2}\right)$$

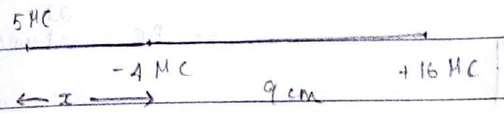
$$\frac{q}{4\pi\varepsilon_0}\left(\frac{l^2+l^2}{l^4}\right) \Rightarrow \frac{q}{4\pi\varepsilon_0}\frac{2l^2}{l^4}$$

$$\frac{q}{4\pi\varepsilon_0}\frac{2}{l^2}$$

or $\dfrac{2q}{4\pi\varepsilon_0 l^2}$ oppo. to the direction of dipole moment.

Q. what should be the position of charge, $q = 5\mu C$ for it to be in equil^m on the line joining 2 charges $q_1 = -4\mu C$ & $q_2 = 16\mu C$ separated by 9 cm? Will the position change for any other value of charge q?

Net force = 0.



$$F_1 = F_2$$

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\times5}{x^2} \qquad F_2 = \frac{1}{4\pi\varepsilon_0}\frac{5\times16}{(9+x)^2}.$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}}\frac{20}{x^2} = \cancel{\frac{1}{4\pi\varepsilon_0}}\frac{80}{(9+x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(9+x)^2}$$

$$\frac{1}{x} = \frac{2}{9+x}$$

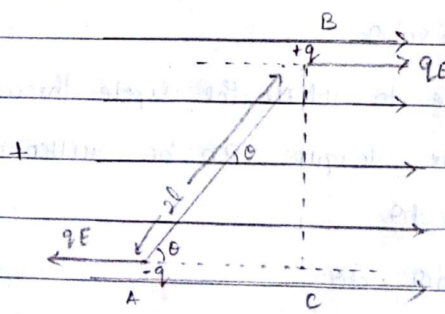
$$9+x = 2x$$

$$x = 9\,cm$$

No change in the position

Torque on a dipole kept in a uniform electric field :-

Consider an electric dipole consisting of 2 charges $-q$ & $+q$ separated by a dist. $2l$, kept in a uniform electric field of intensity, E at an angle θ as shown in fig.



$E = \frac{F}{q}$

$F = Eq$

Force experienced by the charge $+q$, $F = qE$, along the direction of E

Force experienced by the charge $-q$, $F = qE$, ~~along~~ oppo. to the direction of E

These 2 forces are equal & oppo. & acting at 2 diff. points.

∴ The net force experienced by the dipole is 0 but it experiences a torque.

We have torque, τ = Force × ⊥r distance.

$\tau = F \times BC$

$= qE \times 2l \sin\theta$

$= PE \sin\theta$

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

From ΔABC

$\sin\theta = \frac{BC}{2l}$

or $BC = 2l \sin\theta$

→ The direction of torque is ⊥r to both $\vec{P}$ & $\vec{E}$

Special cases

i) Let $\theta = 0°$ or $180°$

Then $\tau = PE\sin\theta$

$\underline{\underline{0}}$

ii) Let $\theta = 90°$

Then $\tau = PE \sin 90$

$= \underline{\underline{PE}}$

That is Torque is max when the dipole is kept ⊥r to the field & is min, when it is ∥^{l} to the field.

Note :- When a dipole is kept in a non-uniform electric field, it will experience a net force as well as a torque.

Potential Energy of a dipole kept in uniform electric field

Consider a dipole of dipole moment P, kept in a uniform electric field of intensity, E at an angle θ.

Then torque, $\tau = PE\sin\theta$

The small work done to rotate the dipole through a small angle $d\theta$ against the torque can be written as,

$dw = \tau \times d\theta$

$= PE \sin\theta \times d\theta$

∴ The tot. work done in rotating the dipole from an angle θ_1 to an angle θ_2, w

$$W = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

ie $PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$ $\qquad \int \sin x \, dx = -\cos x$

$PE \left[-\cos\theta \right]_{\theta_1}^{\theta_2}$

$\therefore \boxed{W = -PE \left[\cos\theta_2 - \cos\theta_1\right]}$

Let $\theta_1 = 90^\circ$ & $\theta_2 = \theta$

$\therefore \quad W = -PE(\cos\theta - \cos 90)$

$W = -PE\cos\theta$

The work done is stored as the PE of the dipole

$\therefore \boxed{U = -PE\cos\theta}$

$\boxed{U = -\vec{P}\cdot\vec{E}}$

Special cases :-

i) Let $\theta = 0^\circ$

Then, $U = -PE\cos 0$

$= -PE$ ie. the energy is min. & the dipole is said to be in the stable equil^m position.

ii) Let $\theta = 90^\circ$

Then, $U = -PE\cos 90^\circ$

$= 0$

iii) Let $\theta = 180^\circ$

Then, $U = -PE\cos 180$ ie the energy is max & the dipole is said to be

$= +PE$ in unstable equil^m position.

A pendulum bob of mass 8mg & carrying charge of 3×10^{-8} C, placed in a horizontal electric field. It comes to equil^m position at an angle of 37° the vertical. Calculate the intensity of electric field.

Net force = 0. (To keep it at an angle of 37)

These should be a force in oppo direction to keep it in that position

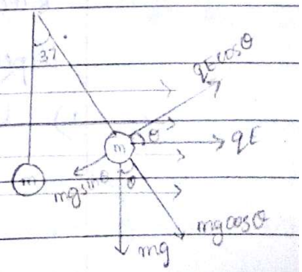
$\therefore mg\sin\theta = qE\cos\theta$

$mg\sin\theta = qE\cos\theta$

$8\times10^{-6} \times 10 \times 0.75 = 3\times10^{-8}\times E$

$E = \dfrac{8\times10^{-5}\times0.75}{3\times10^{-8}}$

$= 2\times10^{3}$ N

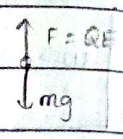


Q. An oil drop of mass m carrying charge $-Q$ is to be held stationary in the gravitational field of earth. What is the direction & magnitude of electric field required for this purpose?

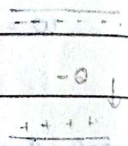
$$mg = QE$$

$$E = \frac{mg}{Q}$$

we need a force upward to keep it at rest.



if considering Electric field is upward

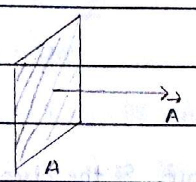


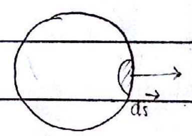
The field is acting in downward direction

Area Vector :-

A vector which represents an area is called the area vector.

→ An area vector is directed ⊥r to the area.

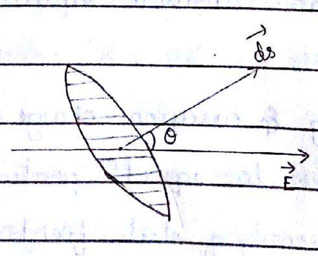




Electric Flux :- The total no. of electric field lines passing normal to a surface is called the electric flux. It can be measured as the dot product of intensity of electric field ~~and~~ ($\vec{E}$) and the area ($\vec{ds}$)

i.e. $d\phi = \vec{E} \cdot \vec{ds}$

$$\boxed{d\phi = E\,ds\cos\theta}$$



Electric flux is a scalar quantity & its unit is Nm^2/C

Special cases :-

i) Let $\theta = 0°$

then $d\phi = E\,ds\cos 0$

$= E\,ds$

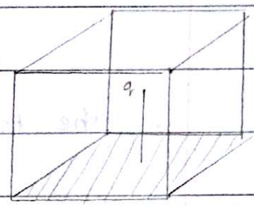
ii) Let $\theta = 90°$

Then $d\phi = E ds \cos 90°$

$= \underline{0}$

Note :- The total electric flux, $\phi = \int \vec{E} \cdot \vec{ds}$

Q. A point charge, q is kept at a height of 5cm from the centre of a square of side 10 cm as shown in fig. Find the total electric flux passing through the square.



Flux through the 6 faces of the cube,

$$\phi = \frac{1}{\varepsilon_0} \times q$$

Flux through the square (1 face)

$$= \phi' = \frac{1}{\varepsilon_0 \times 6} \times q$$

$$\phi' = \frac{q}{6\varepsilon_0}$$

GAUSS THEOREM IN ELECTROSTATICS

i) Gaussian Surface :- It is an imaginary closed surface drawn around a charge

→ It can have any shape

→ It never touches the charge

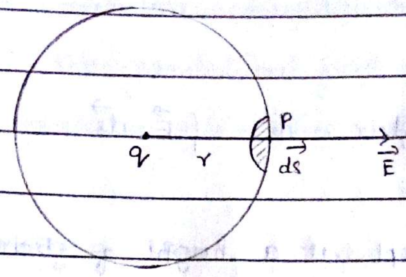
ii) Gauss theorem :- The theorem states that the total electric flux passing through any closed surface is equal to $1/\varepsilon_0$ times the net charge enclosed by the surface.

ie $\boxed{\phi = \frac{1}{\varepsilon_0} \times q}$, where q is the net charge enclosed by the surface.

Proof :- Consider a point P at a distance r from a point charge q. The intensity of electric field at P is given by,

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

Fig. shows a spherical Gaussian surface with q as the centre & r as radius



ds is a small area around the point p.

Electric flux through ds, $d\phi = \vec{E} \cdot ds$

$= E ds \cos\theta$

$= E ds \quad (\because \theta = 0)$

$\therefore$ The total electric flux through the closed surface,

$$\phi = \int d\phi$$

$$\phi = \int E ds$$

$$\phi = E \oint ds$$

$$\phi = \frac{1}{4\pi\varepsilon_0} \times 4\pi r^2$$

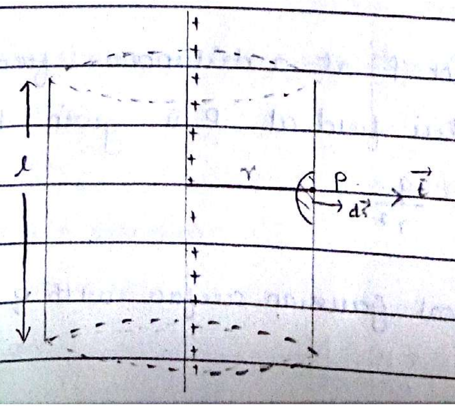
$$\boxed{\phi = \frac{1}{\varepsilon_0} \times q}$$

Applications of Gauss theorem :-

i) Intensity of Electric field due to a line charge :-

Consider an infinitely long line charge of linear charge density (Charge per unit length), λ.

P is a point at a distance, r from the line charge. To find the intensity of electric field at P, we have to consider a cylindrical gaussian surface of radius r & length l as shown in the fig.



The flux through the upper & lower faces of the cylinder is 0 (Since $\theta = 90^\circ$)

$\therefore$ The total flux through the cylinder is the flux through the curved surface only.

ds is a small area around the point P.

$\therefore$ Flux through ds, $d\phi = \vec{E} \cdot \vec{ds}$

$$= E\,ds \cos\theta$$

$$d\phi = E\,ds \quad (\because \theta = 0^\circ)$$

$\therefore$ The total flux through the cylinder, $\phi = \oint d\phi$

$$= \oint E\,ds$$

$$= E \oint ds$$

$$\phi = E \times 2\pi r l \rightarrow ①$$

Net charge enclosed by the gaussian surface, $q = l\lambda \rightarrow ②$

Acc. to Gauss theorem,

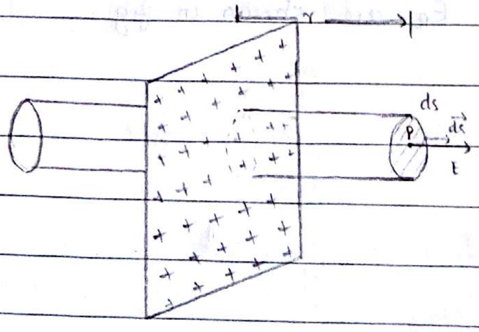
$$\phi = \frac{1}{\varepsilon_0} \times q$$

$$E \times 2\pi r l = \frac{1}{\varepsilon_0} \times q$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}}$$

ii) Intensity of Electric field due to a plane sheet of charge :-

Consider an infinitely long plane sheet of charge of surface charge density (charge per unit area), σ. P is a point at a distance r from the sheet. Fig. shows a cylindrical gaussian surface of end face area, ds.



The flux through the curved surface of the cylinder is 0 ($\because \theta = 90^\circ$)

$\therefore$ The total flux through the Gaussian surface is the sum of the flux through the 2 end faces:

$$\therefore \phi = 2\vec{E}\vec{ds}$$

$$= 2 \times E ds \cos\theta$$

$$\phi = 2Eds \quad (\because \theta = 0^\circ) \rightarrow ①$$

Net charge enclosed by the Gaussian surface,

$$q = \sigma ds \rightarrow ②$$

Acc. to Gauss theorem,

$$\phi = \frac{1}{\varepsilon_0} \times q$$

$$2Eds = \frac{1}{\varepsilon_0} \times \sigma ds$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

From the above result, it is clear that the electric field is independent of distance r.

iii) Intensity of electric field due to 2 ‖le plane sheets of charge

Consider 2 +vely charged ‖le plates, A & B with surface charge densities, σ_A & σ_B resp.

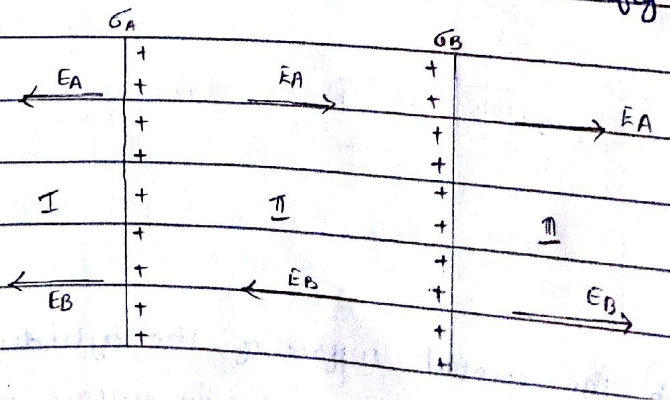
Then, the field due to A,

$$E_A = \frac{\sigma_A}{2\varepsilon_0}$$

Field due to B,

$$E_B = \frac{\sigma_B}{2\varepsilon_0}$$

The directions of E_A & E_B are shown in fig.



Region I

Both E_A & E_B are -ve

$\therefore E = -E_A + -E_B$

$-\frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0}$

$= \frac{-1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

Region II

E_A is +ve & E_B is -ve

$\therefore E = E_A + -E_B$

$\frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0}$

$= \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$

Region III

Both E_A & E_B are +ve

$\therefore E = E_A + E_B$

$\frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0}$

$= \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

Special case :- Let the 2 plates be given equal & oppo. charge

ie, if $\sigma_A = \sigma$

then $\sigma_B = -\sigma$

$\therefore$ Region I :-

$\frac{-1}{2\varepsilon_0}(\sigma - \sigma)$

$= 0$.

Region II

$\frac{1}{2\varepsilon_0}(\sigma - (-\sigma))$

$= \frac{2\sigma}{2\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$

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Region III

$\frac{1}{2\varepsilon_0}(\sigma + \sigma)$

$= 0$

From the above results, it is clear that if the 2 plates are given equal & oppo. charges, then the field will exist only in b/w the plates.

iv) Intensity of Electric field due to a uniformly charged spherical shell :

Consider a uniformly charged spherical shell of radius R & charge q. Then its surface charge density, $\sigma = \frac{q}{4\pi R^2}$

P is a point at a distance r from the centre of the sphere.

Case I :- Let $r < R$

ie the point P lies inside the shell

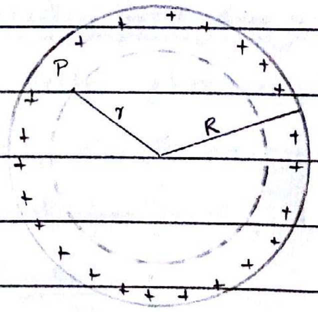


Fig. shows a spherical gaussian surface of radius r

Acc. to gauss theorem, $\phi = \frac{1}{\varepsilon_0} \times$ net charge

Since the net charge enclosed by the gaussian surface is 0,

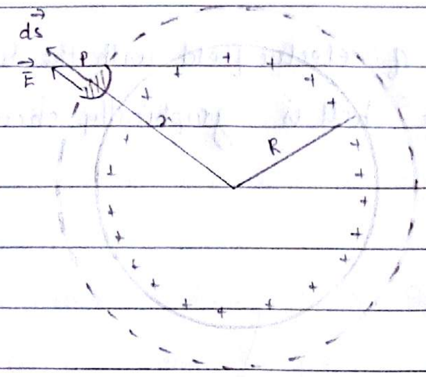
$\phi = 0$

$\therefore E = 0$

ie the electric field inside the shell is zero.

Case II :- Let $r > R$

ie the point P is outside the shell



Flux through the gaussian surface, $\phi = \oint \vec{E}.\vec{ds}$

$= \oint E ds \cos\theta$

$= \oint E ds \quad (\because \theta = 0^\circ)$

$= E \oint ds$

$= E \times 4\pi r^2$

Net charge enclosed by the surface $= q$

Acc. to gauss theorem, $\phi = \frac{1}{\varepsilon_0} \times$ net charge

ie $E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \times q$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}$$

But $q = \sigma \times 4\pi R^2$

$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \frac{\sigma 4\pi R^2}{r^2}$

$$\boxed{E = \frac{\sigma}{\varepsilon_0} \times \frac{R^2}{r^2}}$$

Case III :- Let $r = R$

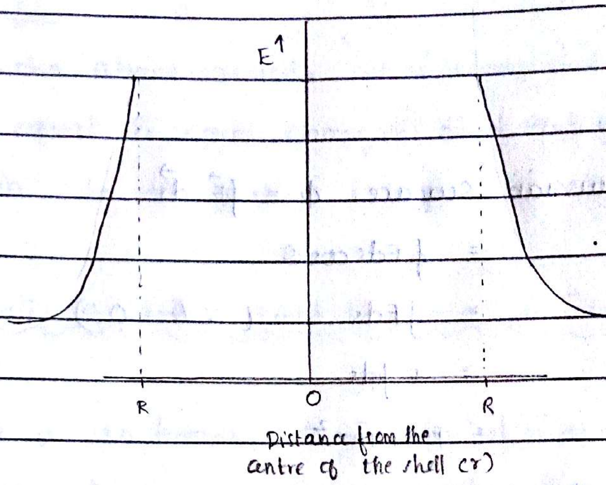
ie point P is on the surface

Then $E = \frac{\sigma}{\varepsilon_0} \times \frac{R^2}{R^2}$

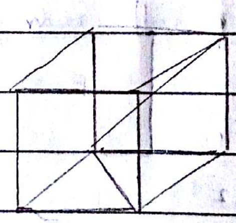
$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

From the above results, it is clear that the intensity of electric field is max. on the surface, 0 inside and decreases when going away from the surface.

The variation of intensity of electric field with the distance from the centre of the shell is graphically shown in the fig.



Q. A line charge of linear charge density λ is arranged inside a cube of side l such that max. electric flux is passing through the cube. Find the flux.



From $\triangle BDC$,

$$l^2 + l^2 = 2l^2$$

From $\triangle ADC$,

$$l^2 + 2l^2 = 3l^2$$

$$= \sqrt{3}l$$

Net charge, $q = \lambda \times \sqrt{3}l$

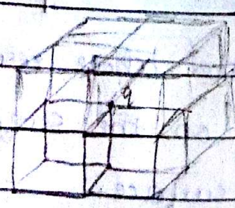
$$\therefore \phi = \frac{1}{\varepsilon_0} \times q$$

$$= \frac{1}{\varepsilon_0} \times \sqrt{3}l\lambda = \frac{\sqrt{3}l\lambda}{\varepsilon_0}$$

Q. A charge q is kept at one of the corner of a cube of side a. Find the electric flux passing through

i) Through the cube

ii) One ~~phase~~ face of the cube

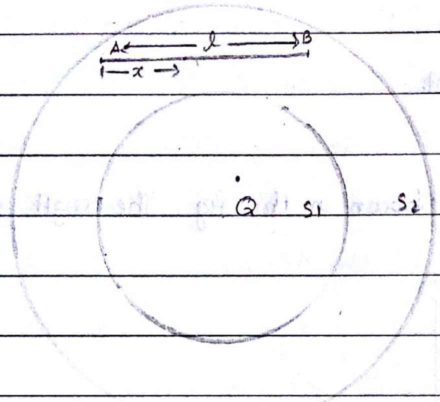


i) Flux through 8 cubes, $\phi = \frac{1}{\varepsilon_0} \times q$

$\therefore$ Flux through 1 cube, $\phi' = \frac{q}{8\varepsilon_0}$

$\therefore$ Flux through 1 face $= \phi = \frac{q}{8\varepsilon_0} \times \frac{1}{3} = \frac{q}{24\varepsilon_0}$

Q. Fig shows 2 conc. spheres S_1 & S_2. AB is a line charge of linear charge density, λ, where $\lambda = kx$. x is the dist. of the line charge measured from the end A. Find the flux through S_1 & S_2.



In S_1:

$$\phi = \frac{1}{\varepsilon_0} \times Q.$$

In S_2.

dx

$dq = dx\,\lambda$ — as x increases, λ changes so λ is not a constant

$dq = dx \times kx$.

$$q = \int_0^l dx\, kx$$

$$q = k\int_0^l x\,dx$$

$$q = k\left[\frac{x^2}{2}\right]_0^l$$

$$q = \frac{1}{2}kl^2.$$

$$\phi = \frac{1}{\varepsilon_0}\left(\frac{1}{2}kl^2 + Q\right)$$

A dipole consisting of 2 charges $-2\mu C$ & $+2\mu C$ separated by a distance of 6cm is kept inside of cylinder of $l = 10$ cm & $r = 2$ cm. Find the flux through the cylinder.

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$$\phi = \frac{1}{\varepsilon_0} \times q$$

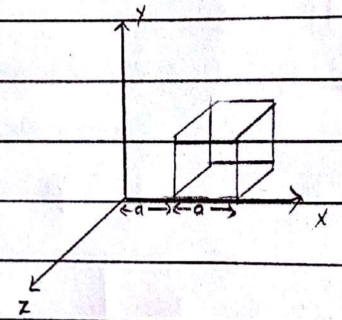
Net charge enclosed = 0.

$\therefore \phi = 0$

Q. A charge is enclosed by a sphere of radius r. The flux through the sphere is ϕ. Find the new ϕ when the radius of the sphere is doubled.

Flux remains constant.

Q. A cube is arranged as shown in the fig. The length of the cube is a



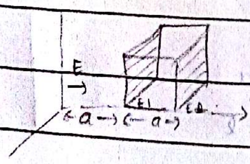
An Electric field is given by $\vec{E} = 2x\hat{i}$ N/C. Find

i) The total electric flux passing through the cube.

ii) The net charge enclosed by the cube.

i) $\phi = E ds$

$= EA$



because it is a func^n of x, E keeps chang as x increases

$\phi_1 = E_1 a^2$

$2x \times a^2$

$2a \times a^2 = 2a^3$

$a = x$.

$\phi_2 = E_2 a^2$

$= 2 \times 2a \times a^2$

$= 4a^3$

$\therefore$ Net $\phi = \underline{2a^3}$

ii) $\phi = \frac{1}{\varepsilon_0} q$

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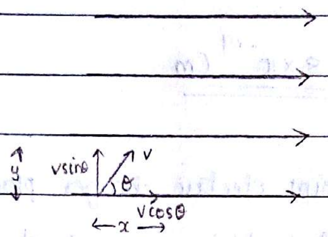
$q = \phi \varepsilon_0$

$\therefore q = 2a^3 \varepsilon_0$

A charge q is entering a uniform electric field of intensity E with a velocity v. at an angle θ. Show that the path of the charge is a parabola



x	y
$u = v\cos\theta$	$u = v\sin\theta$
$t = t$	$t = t$
$s = x$	$s = y$
$F = qE$	$a = 0$

$s = ut + \frac{1}{2}at^2$

$y = v\sin\theta\, t$

$t = \dfrac{y}{v\sin\theta}$

$F = ma$

$ma = qE$

$a = \dfrac{qE}{m}$

$s = ut + \frac{1}{2}at^2$

$x = v\cos\theta\, t + \dfrac{1}{2} \times \dfrac{qE}{m} t^2$

$x = \cancel{v}\cos\theta \times \dfrac{y}{\cancel{v}\sin\theta} + \frac{1}{2} \times \dfrac{qE}{m} \dfrac{y^2}{v^2\sin^2\theta}$

$\cot\theta\, y + \frac{1}{2} \dfrac{qE}{m} \dfrac{y^2}{v^2\sin^2\theta}$ (for an angle of proj θ is const)

$x = ay + by^2$

The above equ^n represents a parabola

Q. An electric dipole when held at 30° with respect to a uniform electric field of 10^4 N/C experienced a torque of 9×10^{-26} Nm. Calculate the dipole moment.

$\tau = PE\sin\theta$

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$$9 \times 10^{-26} = P \times 10^{4} \sin 30$$

$$\therefore P = \frac{9 \times 10^{-26}}{10^{4} \times \frac{1}{2}}$$

$$= \frac{9 \times 10^{-26}}{5 \times 10^{3}}$$

$$= \frac{9}{5} \times 10^{-29} \Rightarrow \underline{1.8 \times 10^{-29}\ Cm}$$

$$5\overline{)9}\ 1.8$$
$$5$$
$$40$$

Q. Force of attraction b/w 2 point electric charges placed a distance d in a medium is F. what distance apart should there be kept in the same medium so that Force b/w them becomes F/3.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

$$F' = \frac{F}{3}$$

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(d')^2}$$

$$F' = \frac{F}{3}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d_1^2 \times 3}$$

$$\frac{F}{F'} = \frac{\cancel{\frac{1}{4\pi\varepsilon}}\ \frac{\cancel{q_1 q_2}}{d^2}}{\cancel{\frac{1}{4\pi\varepsilon}}\ \frac{\cancel{q_1 q_2}}{d'^2 \cancel{\times 3}}}$$

$$\frac{F}{F'} = \frac{\cancel{3}d'^2}{d^2}$$

$$\frac{\cancel{F}}{\frac{\cancel{F}}{3}} = \frac{(d')^2}{d^2}$$

$$\frac{3d'^2}{d^2} = 1$$

$$3d^2 = \cancel{3}d'^2$$